Path Planning maximising Human Comfort for Assistive Robots

Paolo Bevilacqua, Marco Frego, Enrico Bertolazzi, Daniele Fontanelli, Luigi Palopoli, Francesco Biral

***Abstract*— Motion planning for service robots used for human assistance in complex environments is a challenging task with multifaceted requirements. On the one hand, the generated path has to avoid obstacles and satisfy the user requirements. On the other, the generated path has to be perceived as comfortable by the user. An efficient solution to this difficult problem is based on a separation of concerns between two subproblems: 1. generating a sequence of waypoints that connects the source to the destination, 2. generating a path that joins the waypoints and complies with the dynamic constraints. In this paper, we concentrate on the second problem with an explicit focus on how to account for users' comfort. The result is accomplished by solving an optimal control problem that inherently generates the smooth trajectories required by the user. The cost functions used in the optimisation problem allows to take different dimensions of the user's comfort into consideration. Our technique lends itself to an efficient numeric solution and can be easily applied to different algorithms for waypoint generation.**

I. Introduction

In the past few years, several research projects on both sides of the Atlantic Ocean and in Japan have targeted a particular application of service robotics: assisting older adults or disabled users in their navigation of public spaces. Such efforts have generated interesting proposals for smart wheelchairs [1], [2] or intelligent walkers for older adults [3], [4], [5]. Key to the acceptability of these technological solutions are the level of comfort of the trajectories planned to travel across crowded environments, their safety [6] and and their compliance with social rules [1], [7].

The challenging task of navigation in human environments is typically solved using a two phases approach: 1) generation of a sequence of waypoints to reach the destination, 2) generation of a path joining the waypoints that optimises the robot manoeuvres. This decomposition reflects a sensible separation of concerns between finding a path that avoids obstacles and satisfies the user requirements, and generating a curve that respects dynamic constraints.

A plethora of solutions exist in the literature for the generation of waypoints. A classical approach is the generation of a graph representing the environment (e.g., using quad-tree decompositions [8]) followed by the application of the Dijkstra's algorithm to find the shortest path [9]. An established trend is toward the use of randomised algorithms such as the Rapidly-exploring Random Trees (RRT) [10], or its recent RRT* [11] evolution. The generation of waypoints can easily account for such constraints as "plan a path that stay close most of the times to benches" by using high level planning languages [12] or modified versions of Dijkstra's solution [13]. What is more, a global plan consisting of a sequence of waypoints can be "locally" fine-tuned using contextual information collected on the ground (e.g., presence of by-standers). Important examples are the Risk RRT [14], based on RRT*, stochastic approaches making use of statistical models to predict the motion of humans nearby the robot [15], [16], or elastic bands to adapt a plan to the presence of obstacles [17].

In this paper, we take on the problem of generating an optimal path joining the waypoints, assuming that the latter have been generated already by the most convenient solution for the problem at hand. The generation of the optimal path has been traditionally looked at from the perspective of the vehicle dynamics. From a practical view point, it is possible to plan a path in an environment with any standard planner (such as RRT* [11] or the A* algorithm [18]) and then run on it a simulation of the vehicle smoothing the curvatures. This is exactly the line followed first in [17] and by a number of similar papers that followed.

The point we make in this paper is that effective *motion planning algorithms for assistive robots has to explicitly consider user comfort as a first class citizen in path generation*. User comfort (or lack of) has been historically studied for ground vehicles, such as cars or trains, and for other transportation systems, and it is widely recognised that the discomfort increases with body accelerations and jerk [19], [20], [21]. Such results have been confirmed by neuroscience results on grasping or humans' arms motion [22]. When it comes to evaluating the comfort of trajectories of mobile robots, the jerk and the acceleration are obviously related to the curvature of the path and to possible discontinuities in it.

Most of the vehicles used for service robotics are unicycles or car-like vehicles (or simplified variations). The latter have evident advantages for comfort since the type of curves they move along are smooth (segments of clothoid) and have continuous curvature. What is more, optimal solutions for car–like vehicles are very efficient to find using semi-analytic techniques [23], [24], [25]. Finally, direct observations made by Laumond et al. [26] suggest that humans tend to move according to a kinematic non-holonomic model, which bears

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a close resemblance with a simplified car-like model.

All this considerations suggested us to set up path planning as an optimal control problem *a la* Pontryagin for a simplified car–like. The boundary conditions for the problem are given by the waypoints, and different functionals related to the path curvature or to its length can be used to express the user's comfort. This is possible even if our vehicle is actually a unicycle, by the addition of "artificial" dynamics that constrain its motion along the same trajectories as a car–like vehicle. Typical solutions in the literature to this type of problems are based on the use of a finite set of primitives. For instance Fraichard et al. [27] use circular arcs, tangent straight lines or continuous curvature segments to generate a path, but they produce discontinuous curvatures in the switching points, with a poor resulting comfort. Our approach is based on the use of segments of clothoid as geometric primitives. Such curves are arguably the outcome of our optimal control problem for our non-holonomic model. Moreover, using the semi-analytic solution proposed in [23], such a solution can be found very efficiently. In this paper, we show the application of this algorithm by using both the waypoints generated by RRT* and by the Dijkstra algorithm [9].

The paper is organised as follows. Section II describes the platform model, the comfort indices and the problem to be solved. Section III presents the model formulated in curvilinear coordinates and details the proposed solution, which is finally presented with simulation comparisons in Section IV. Conclusions draw the reader attention to the advantages of the proposed method and to the future developments and extensions.

II. Modelling and Problem Statement

The use of robotic platforms to help older adults navigate in complex environments is commonly regarded as an effective means to extend their mobility and, ultimately, to improve their health conditions. These assistive platforms aims at offering several types of cognitive and physical support, such as navigation. The basic idea is to endow a classic physical support system, such as a standard rollator or a wheelchair, with navigation capability. From a strict control view-point, the large majority of these mechanical platforms can be seen as unicycle–like or car–like vehicles. The objective of this section is to reduce this class of support systems to a single model, to derive a suitable set of discomfort indices to be minimised and to formally derive the optimal problem at hand.

A. Platform Model

With reference to Figure 1, let $\langle W \rangle = \{O_w, X_w, Y_w, Z_w\}$ be a fixed right-handed reference frame, whose plane $\Pi = X_w \times Y_w$ is the plane of motion of the vehicle, Z_w pointing outwards the plane Π and let O_w be the origin of the reference frame. Let $\mathbf{x} = [x, y, \theta]^T \in \mathbb{R}^2 \times S$ be the kinematic configuration of the platform, where (x, y) are the coordinates of the mid–point of the rear wheels axle in Π and θ is the orientation

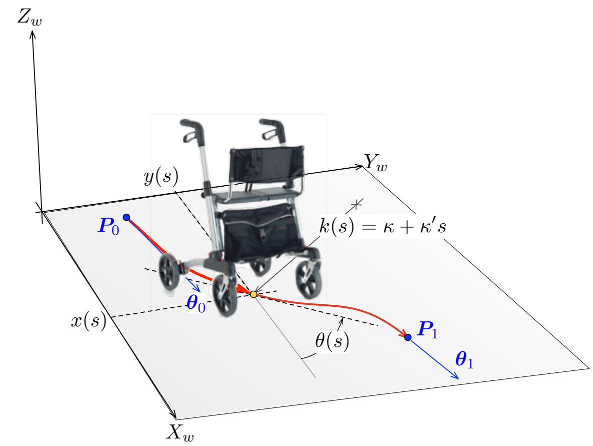


Fig. 1. The reference system for the trajectory planning with clothoidal elements.

of the vehicle w.r.t. the X_w axis (see Fig. 1). Assume that the kinematic model of the mechanical platform can be assimilated to a unicycle–like vehicle, described by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v\cos(\theta) \\ v\sin(\theta) \\ \omega \end{bmatrix} \tag{1}$$

where v and ω are the forward and the angular velocities.

As mentioned in the Introduction, for assistive robots, the accelerations and jerk experienced by the user are determined by the geometry of the path to follow, which needs to have continuous curvature [20]. Since the unicycle model in (1) does not satisfy this requirement, we propose a dynamic extension of the unicycle model, whose kinematic ODEs in the cartesian xy-coordinates are given by:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\bar{\delta}} \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ \bar{\delta} \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \bar{\omega}, \tag{2}$$

where $\bar{\delta}$ can be interpreted as the steering angle and $\bar{\omega}$ its velocity.

The same model can also be applied if our vehicle is natively a car–like vehicle with rear traction, described by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ \tan(\delta)/l \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \omega_\delta, \tag{3}$$

where δ is the actual steering angle, ω_δ is the normalised angular velocity of the steering wheel and $l > 0$ is the wheelbase. In this case, the kinematic model (3) can be easily reduced to the model (2) by using the auxiliary control input

$$\bar{\omega} = \left(\delta^2 + 1\right) \omega_\delta,$$

and assuming that $\tan(\delta) \approx \bar{\delta}$ (see [23], [27]).

Model (2), which is the baseline for our developments, deserves some additional discussion. First, according to [26], the locomotion trajectories of human beings are well approximated by the optimal solution of (2), the claim being validated by the analysis of a large number of recorded trajectories of groups of people moving in a real environment. Secondly, [27] shows that the car-like time optimal trajectory is given by a sequence of *clothoids* (depicted in Section III) whenever the velocity v is considered constant, while [23] shows that even with a varying velocity v, the curvature of the path for model (2) can be safely approximated with a piecewise linear curvature, which corresponds to a clothoid or to a sequence of clothoids (see Figure 1).

B. Comfort

The goal-directed locomotion analysis of humans considering different sensory inputs, for example visual or vestibular, and the interaction between the head and the torso or the eyes and the limbs has received a constant attention. However, only recently has a deep understanding of the human trajectories become a relevant research issue. An important research direction is to seek the functional that a person optimises when she walks and the related suitable dynamic system to describe it. While convincing results on the latter problem are available in the form of a system of differential equations that governs the human locomotion, the optimised functional is much less obvious.

Using model (2), [26] shows that the longitudinal velocity can be considered constant for a human navigation task and that the functional optimised along the trajectories is the minimum square of the jerk. The jerk is defined as the derivative of the acceleration and seems to match the experimental evidences very satisfactorily. The expression of the jerk in terms of the velocity v and the curvature k is $j = v^2\dot{k}$. For the optimal curve, i.e., the clothoid, the curvature is a linear function of the arc-length s, and is expressed by $k(s) = \kappa + \kappa' s$, for two real coefficients κ and $\kappa' = d\kappa/ds$. Thus we specialise the expression of the jerk combining the information of the curvature, which yields $j = v^2\kappa'$. By assuming the velocity to be constant, according to [26], we can consider as a measure of the trajectory jerk the square of κ'. As a consequence, a possible cost index that can be considered for the human comfort is the minimisation of the jerk, i.e.

$$T_1 = f_1(\kappa'^2),$$

where $f_1(\cdot)$ is a suitable function to be defined. While the jerk is obviously relevant, other comfort indices could be considered as well. From the previous analysis, a minimisation of the overall curvature can also be a suitable cost index to be minimised to increase the comfort, which can be denoted as:

$$T_2 = f_2(\kappa, \kappa').$$

Finally, according to [20], the minimisation of the path length could be considered as a relevant comfort index, i.e.

$$T_3 = f_3(\kappa, \kappa'),$$

where T_3 is equal to the time optimal path, the velocity v being assumed constant.

C. Problem Formulation

We are now in condition to produce a formal statement of the problem addressed in the paper. *Given a path described by a sequence of waypoints in the selected environment, interpolate them with a clothoid spline with continuous curvature that minimises the selected cost index (i.e., T_1, T_2 or T_3).*

The sequence of waypoints points may be obtained with various methods, such as Dijkstra [9] and A* [18] on quadtrees decompositions [8] or using RRT* [11]. The main point of this choice is that the proposed method can compute a non smooth path very quickly, thus they are very suitable in the case of dynamic environments or socially aware navigation. Nevertheless, the paths thus synthesised are given only by a sequence of points, while a smooth trajectory that minimises a discomfort index is instead necessary.

III. Proposed Solution

The curves that can be obtained by the kinematic model (2) are clothoids, hence we fit a clothoid spline through the given sequence of waypoints. A clothoid arc can be represented as a parametric function of the arc-length s via the Fresnel Integrals [28], and is characterised by six real parameters: (x_0, y_0), the initial point, θ_0, the initial angle, κ, κ' the curvatures and the length L. The space coordinates of a point on the clothoid at arc-length s, the corresponding angle and curvature are given by:

$$\begin{aligned}
x(s) &= x_0 + \int_0^s \cos\left(\frac{1}{2}\kappa' t^2 + \kappa t + \theta_0\right) \mathrm{d}t && (4)\\
y(s) &= y_0 + \int_0^s \sin\left(\frac{1}{2}\kappa' t^2 + \kappa t + \theta_0\right) \mathrm{d}t && (5)\\
\theta(s) &= \frac{1}{2}\kappa' s^2 + \kappa s + \theta_0, && (6)\\
k(s) &= \kappa' s + \kappa. && (7)
\end{aligned}$$

A useful mode to compute a clothoid is to use the Fresnel Generalised Integrals (FGI), which are defined as follows:

$$\begin{aligned}
X_j(a,b,c) &= \int_0^1 \tau^j \cos\left(\frac{a}{2}\tau^2 + b\tau + c\right) \mathrm{d}\tau,\\
Y_j(a,b,c) &= \int_0^1 \tau^j \sin\left(\frac{a}{2}\tau^2 + b\tau + c\right) \mathrm{d}\tau,
\end{aligned}$$

which can be plugged into (4) and (5) leading to:

$$\begin{aligned}
x(s) &= x_0 + s\, X_0(s^2\kappa', s\kappa, \theta_i)\\
y(s) &= x_0 + s\, Y_0(s^2\kappa', s\kappa, \theta_i).
\end{aligned}$$

A clothoid spline is a C^2 curve whose curvature is a continuous piecewise linear function of the arc-length, i.e., a sequence of n clothoids. Such a curve $\mathcal{C}$ is given by a finite collection of real parameters $0 = s_0 < s_1 < \ldots < s_n$ and $(\boldsymbol{p}_i, \theta_i, \kappa_i, \kappa_i')$ with $\boldsymbol{p}_i = (x_i, y_i) \in \mathbb{R}^2$ and $i = 0, 1, \ldots, n-1$. For each i, $\mathcal{C}$ is a clothoid over $[s_i, s_{i+1}]$. A C^2 spline has the parameters that satisfy the following continuity conditions between two consecutive segments i and $i+1$:

$$\begin{aligned} H_{i,0} = &\quad x_i + L_i X_0\big(\kappa_i' L_i^2, \kappa_i L_i, \theta_i\big) - x_{i+1} = 0, \\ H_{i,1} = &\quad y_i + L_i Y_0\big(\kappa_i' L_i^2, \kappa_i L_i, \theta_i\big) - y_{i+1} = 0, \\ H_{i,2} = &\quad \frac{1}{2}\kappa_i' L_i^2 + \kappa_i L_i + \theta_i - \theta_{i+1} = 0, \\ H_{i,3} = &\quad \kappa_i' L_i + \kappa_i - \kappa_{i+1} = 0, \end{aligned} \tag{8}$$

where $L_i = s_{i+1} - s_i > 0$ and $i = 0, 1, \ldots, n-1$. The first two conditions mean that segments are joined, the third condition implies equality of the tangent at the node, the last condition is the curvature continuity. It is convenient to collect all the constraints (8) in vector form, therefore we introduce the function $\boldsymbol{H}_i(\theta_i, \kappa_i, \kappa_i', L_i, \theta_{i+1}, \kappa_{i+1}) = \mathbf{0} \in \mathbb{R}^4$ for $i = 0, 1, \ldots, n-1$, which represents a system of four equations in six unknowns. As a consequence, the nonlinear system of constraints (8) is not completely determined and this opens to the possibility of minimising different cost functionals. We use the solution of the G^1 Hermite Interpolation Problem with clothoids presented in [28] that allows to solve the first three equations of (8), and reduces the previous nonlinear system from 4 to 1 equation per clothoid segment. The G^1 algorithm takes as input the initial (x_i, y_i, θ_i) and final $(x_{i+1}, y_{i+1}, \theta_{i+1})$ points and angles and computes the i–th clothoid curvature parameters κ_i, κ_i' and its length L_i. Since the trajectory is specified by a sequence of fixed points, by changing the unknown angles θ_i and θ_{i+1} it is possible to ensure continuity between the clothoid curvature of the previous $i-1$–th clothoid segment and the i–th clothoid. This leads to computationally efficient solutions.

We are now in a position to better clarify the comfort indices presented in Section II-B. We are interested in three different functionals $T_j(\theta_0, \theta_1, \ldots, \theta_n)$, $j = 1, 2, 3$, depending only on the unknown angles θ_i that are:

- Minimise the jerk, which is, in this case, equivalent to the minimisation of the variation of the curvature:

$$T_1 = \sum_{i=0}^{n-1} \kappa_i'(\theta_i, \theta_{i+1})^2; \tag{9}$$

- Minimise the integral of the curvature squared,

$$T_2 = \sum_{i=0}^{n-1} \int_0^{L_i} \big(\kappa_i(\theta_i, \theta_{i+1}) + s\kappa_i'(\theta_i, \theta_{i+1})\big)^2 \, \mathrm{d}s \tag{10}$$

- Minimise the total length of the curve:

$$T_3 = \sum_{i=0}^{n-1} L_i(\theta_i, \theta_{i+1}). \tag{11}$$

Each functional (9), (10), (11) with the constraints (8) defines an optimal problem

$$\begin{aligned} &\text{Minimise} && T_j(\theta_0, \theta_1, \ldots, \theta_N) \\ &\text{Subject to} && \boldsymbol{H}_i(\theta_i, \kappa_i, \kappa_i', L_i, \theta_{i+1}, \kappa_{i+1}) = \mathbf{0}. \end{aligned}$$

where $j = 1, 2, 3$ and $i = 0, \ldots, n-1$, that can be efficiently solved using NonLinear Programming (NLP). Notice that due to the periodicity of the involved trigonometric functions there can be many local minima. This correspond to (undesired) loops in the trajectory that can be avoided with the provided right guess for the NLP solver. An effective way to compute the NLP solution is to use the software IPOPT [29], that requires the derivatives of the target and of the constraints (i.e., gradient and Jacobian). This has been done by analytically differentiating the equations of the G^1 algorithm, i.e., the derivative with respect to the angles θ_i and θ_{i+1}. For space limits, we omit this analytic computation. In [28] was proved existence and uniqueness of the function $\kappa_i'(\theta_i, \theta_{i+1})$ in the appropriate range angle. Limiting the angles θ_i to closed intervals, problems $T_j(\theta_0, \theta_1, \ldots, \theta_n)$ are minimisations of smooth functions on a compact set, thus the solution always exists.

A. Path feasibility

Once the clothoid spline trajectory passing through the set of planned points is synthesised, its feasibility, i.e., the fact that is confined in the free space, has to be carefully considered. The obstacles are assumed to be described by clothoid arcs as well. This assumption is very mild in indoor environments, since straight line segments and arc of circles are clothoids. To check if a clothoid intersects an obstacle, we segment a clothoid, described by the above parameters, on the basis of the travelled space s using segments of angle width at most $\pi/2$. Each clothoid arc has at most one change of sign in the curvature, for $s = -\kappa_i/\kappa_i'$, if this points falls into the interval of the definition of the arc, then we cut the clothoid there in order to have two segments with constant sign curvature. Notice that if both $\kappa_i = \kappa_i' = 0$, the clothoid boils down to a straight line, while if only $\kappa_i' = 0$ the clothoid is an arc of circle. The next step of the segmentation is to divide the previously obtained arcs in equal intervals such that the travelled angle $\pi/2$. The idea is to segment the clothoid and the obstacles on the basis of their curvature and then inscribe each portion in a triangle (see Figure 2 for a visual reference). To this end, given the i–th clothoid, we identify the initial point with $P_{i,0} \triangleq (x_{i,0},\ y_{i,0})$, the final point with $P_{i,1} \triangleq (x_{i,1},\ y_{i,1})$ at abscissa $s^\star$, and we show that the third vertex $P_{i,2} \triangleq (x_{i,2},\ y_{i,2})$ of the triangle $P_{i,0}P_{i,1}P_{i,2}$ is given by

$$P_{i,2} = \left(-\frac{q_0 - q_1}{m_0 - m_1},\ \frac{m_0 q_1 - m_1 q_0}{m_0 - m_1}\right), \tag{12}$$

where $m_0 = \tan(\theta_{i,0})$, $q_0 = y_{i,0} - m_0 x_{i,0}$, $m_1 = \tan(\theta(s^\star))$, $q_1 = y_{i,1} - m_1 x_{i,1}$. The following Theorem ensures that each clothoid can be inscribed in a triangle.

Theorem 1: Consider the clothoid segment obtained as described above, that is, a clothoid of parameters κ_i, κ_i', θ_i of length L_i such that the curvature has constant sign on the interval $(0, L_i)$ and the variation of the angle $\theta(s)$ is less than $\pi/2$, i.e. $\theta(L_i) - \theta(0) \le \pi/2$. Then all the clothoid is contained in the triangle $P_{i,0}P_{i,1}P_{i,2}$.

The proof of the theorem, which is based on purely geometric arguments, is not reported for the sake of brevity.

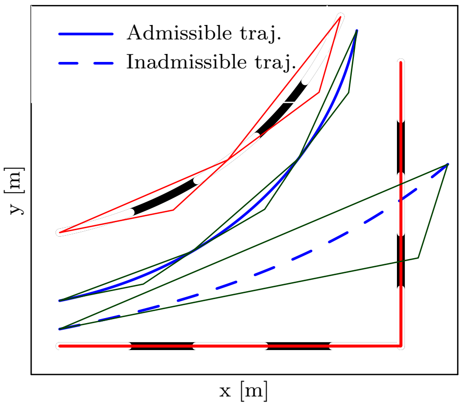


Fig. 2. An example of segmentation with triangles for an admissible (solid) and an inadmissible (dashed) trajectory. The thick dashed lines represents the walls of a portion of a corridor, segmented with triangles as well.

In order to increase the computation performance, the tree of the triangles is built hierarchically using axis-aligned bounding boxes. Each triangle is inscribed in a rectangle with the sides aligned with the axes, and the tree is built recursively, with a top-down method. From a list of such rectangles, the root of the tree is defined as the bounding box containing all the rectangles, then we split the root rectangle at the midpoint of the longest side creating the first two branches of the tree and so on. The collision is then detected computing a standard polygon intersection. In the case of a collision, we simply resample some points of the given sequence and repeat the trajectory construction process.

IV. Results

In this section we propose paths synthesis on the map of the Department of Information Engineering and Computer Science of the University of Trento. The map of the building is represented by a set of polygons corresponding to obstacles and walls. To avoid paths passing too close to these polygons, an offset with a fixed size is generated around them.

The sequence of waypoints joining source and destination has been generated using two different algorithms. The first is based on the decomposition of the map into the well known quad-tree cells [8]. Cells on the boundary between the free space and the obstacles are recursively subdivided into four subcells, until their size is below a fixed resolution parameter. A graph representing free adjacent cells is built, and the shortest path between two nodes is found using the Dijkstra's algorithm [9]. The second algorithm is the Informed RRT* (I-RRT*) [30], a modified version of the RRT* algorithm where the sampling space for feasible points is reduced to an ellipse to speed up the computation. For each sampled point, the corresponding segment is added to the tree only if it is collision free, where the collision is checked using the solution presented in Section III-A. The sequence of points constituting the plan obtained with any of the two is then further processed by removing redundant points aligned on straight lines, and inserting some points before and after each curve, in order to add degrees of freedom for the clothoid spline in the close proximity of a change of direction.

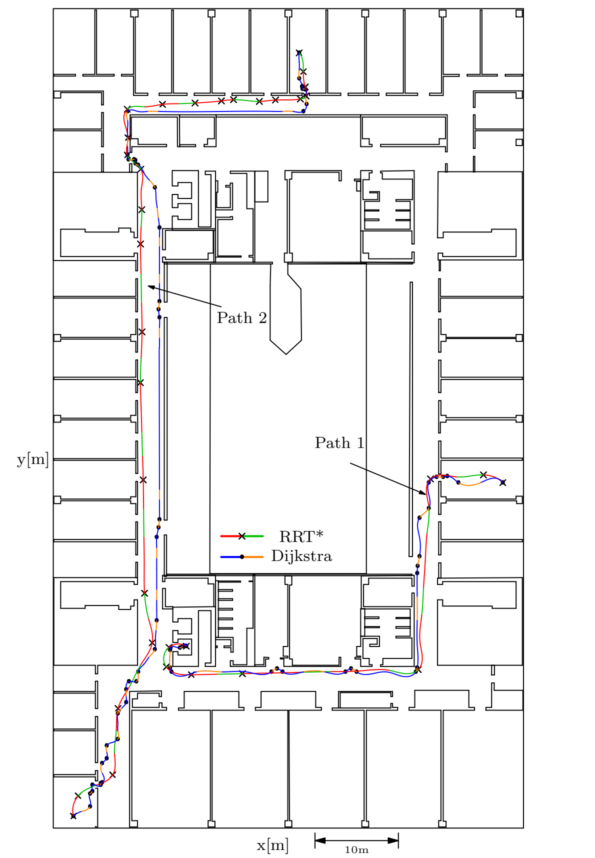


Fig. 3. Comparison of clothoid paths generated with I-RRT* and Dijkstra's algorithm on the map of the Department of Information Engineering and Computer Science of the University of Trento.

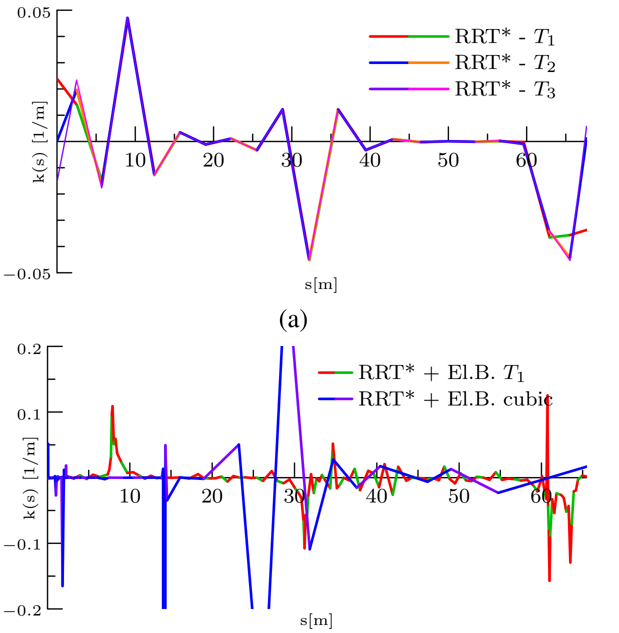
TABLE I

Comparison of different discomfort indices and planning algorithms on the two different paths of Figure 3.

		I-RRT*			Dijkstra		
		Target	Time [s]	Length [m]	Target	Time [s]	Length [m]
Path_1	T_1	0.002	0.90	67.75	4.52	2.39	69.24
	T_2	1.00	1.19	67.66	5.42	2.27	68.46
	T_3	67.66	1.10	67.66	68.44	2.16	68.44
Path_2	T_1	0.19	2.87	116.54	77.79	8.78	121.35
	T_2	1.57	2.89	116.44	25.22	8.20	121.33
	T_3	116.43	2.76	116.43	121.33	8.78	121.33

Figure 3 reports the paths generated using the Dijkstra's algorithm and the I-RRT* minimising the T_1 discomfort index (minimum jerk trajectories). The solution with Dijkstra's algorithm looks less natural with respect to the paths generated using I-RRT*. This is due both to the discrete set of configurations available and to the particular structure of quad-trees, having a higher number of cells along the boundaries of the obstacles. For a quantitative comparison between the different solutions, Table I reports the discomfort index target value, the computation time and the path length for all of the three discomfort indices reported and the two sample paths reported in Figure 3. The solution based on I-RRT* is the one with the best perfromance for all cases.

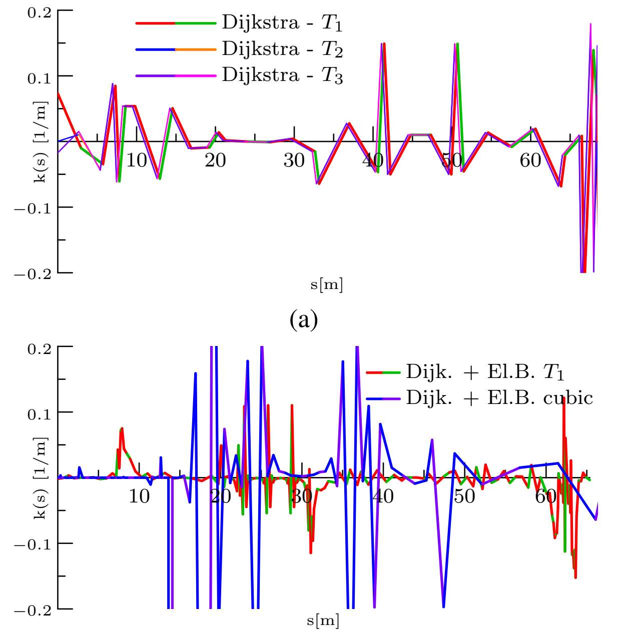
Figures 4-(a) and 5-(a) show the curvature of the paths



(a)

(b)

Fig. 4. (a): Plot of the curvatures for Path 1 of Figure 3 generated with I-RRT* for the different discomfort indices. (b): Path 1 with I-RRT* and elastic bands smoothing for clothoid trajectories (target T_1) or cubic splines.



(a)

(b)

Fig. 5. (a): Plot of the curvatures for Path 1 of Figure 3 generated with the Dijkstra's algorithm for the different discomfort indices. (b): Path 1 with the Dijkstra's algorithm and elastic bands smoothing for clothoid trajectories (target T_1) or cubic splines.

interpolated with the three different cost functionals. The curvatures are overlapping almost everywhere over the path, implying that the constraints imposed by the curvature continuity dominates the path synthesis, except for the initial and the final part of the path. Indeed, the initial and the final curvature are 0 when the adopted functional is T_2 (overall curvature minimisation). Independently from the chosen functional, the range of values of the curvature is much higher for paths generated using Dijkstra, which provides an insight for the results reported in Table I.

The method here proposed has been further compared with a quite popular solution in the literature, which is based on the application of the *elastic bands* (El.B.) [17] to smooth the path, make it more compact and to remove redundant points. This approach is based on the concept of bubbles of free space around discrete configurations composing the path (provided by either Dijkstra's algorithm or I-RRT*), which are moved away from obstacles and toward each other (similarly to an elastic band), by means of virtual forces. The obtained points are then usually interpolated with standard cubic splines. Table II reports a comparison beween the previous clothoid trajectories (No El.B.-T_1) with El.B. with cubics for Path 1 of Figure 3 minimising the trajectory jerk (T_1 discomfort index). It can be seen that the solution with El.B. interpolated with cubics is faster than No El.B.-T_1, but it results into higher lengths and into a much higher discomfort. Albeit a quantitative analysis for the discomfort cannot be easily computed and to prove the flexibility of the proposed approach, the higher discomfort is shown applying the proposed clothoid trajectory generator to the El.B. points (El.B.-T_1). Figures 4-(b) and 5-(b) reports a comparison

TABLE II

COMPARISON OF THE RESULTS WITH AND WITHOUT ELASTIC BANDS FOR PATH 1 AND THE MINIMISATION OF THE JERK T_1, USING BOTH CLOTHOID SPLINES WITH CURVATURE CONTINUITY AND CUBIC SPLINES.

	I-RRT*			Dijkstra		
	Target	Time [s]	Length [m]	Target	Time [s]	Length [m]
No El.B.-T_1	0.002	0.90	67.75	4.52	2.39	69.24
El.B.-cubics	–	0.21	68.54	–	0.305	69.43
El.B.-T_1	3.30	12.67	65.48	10.26	21.31	65.37

of the curvature for the El.B. with cubics and with the clothoids for both the Dijkstra's algorithm and the I-RRT*. It can be seen how El.B.-T_1 produces lower discomfort than the cubic interpolation, that is however higher than the original solution (see Table II). Interestingly, El.B.-T_1 produces shorter paths than No El.B.-T_1.

The proposed solution is reported in pseudo-code in Algorithm 1.

V. Conclusions and Future Work

In this paper we have shown a path panning solution for mobile robots used for navigation assistance for older adults and disabled users. We have used a two tier technique, in which a set of waypoints is first generated using standard algorithms in the literature, and then an optimal path is found optimising the user comfort. The latter goal is achieved by using a nonholonomic car–like mode that naturally generates smooth curves (segments of clothoids). The model is used in the context of an optimal control problem where the cost functional can be related to several parameters expressing different dimensions of the user comfort. In the paper we

Algorithm 1: Generate a spline of clothoids from a given starting position to a given goal using the specified cost

Data: map, $start$, $goal$, $target$
Result: Smooth path from start to goal
function *GeneratePath*
    map ←processMap(map)
    switch $algorithm$ **do**
        case $DIJKSTRA$ **do**
            $quadtree$ ←genQuadtree(map)
            $graph$ ←genGraph($quadtree$)
            $path$ ←dijkstra($graph$, $start$, $goal$)
        end
        case $RRTSTAR$ **do**
            $path$ ←rrtStar(map, $start$, $goal$)
        end
    end
    if $elasticBand$ **then** $path$ ←smoothElasticBand($path$) ;
    else $path$ ←uniformPoints($path$) ;
    return ClothoidInterp($path$, $target$)
end

have shown several combination of this solution with different algorithms that generate the waypoints.

Much is left for future extensions of this work. A first possibility is to relax the constraint that the path touches the waypoints, and allow for the possibility of small deviation using a least square approximation. Another line of research can be the combination of our path planning solution with different algorithms for the generation of the waypoints than the ones considered here. Finally, an ambitious direction could be solving the waypoint generation and the optimal path planning problems at the same time. This possibility holds the promise to further improve the optimal value of the solution and, hence, the perceived comfort.

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